

Research Article

## Self-Consistent Renormalization Theory of Anisotropic Spin Fluctuations in Nearly Ferromagnetic Metals

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**Citation:** Riki o Konno (2024) Self-Consistent Renormalization Theory of Anisotropic Spin Fluctuations in Nearly Ferromagnetic Metals. *Nano Technol & Nano Sci J* 6: 160.

**Received:** July 10, 2024; **Accepted:** July 17, 2024; **Published:** July 20, 2024.

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### Abstract

We investigated the temperature dependence of the inverse of the magnetic susceptibility, the nuclear magnetic relaxation rate, and the  $T$ -linear coefficient of the specific heat in nearly ferromagnetic metals by using the self-consistent renormalization theory of anisotropic spin fluctuations. At low temperatures, the inverse of the magnetic susceptibility has  $T^2$ -linear dependence. In elevated temperatures, the inverse of the magnetic susceptibility has  $T$ -linear dependence. The nuclear magnetic relaxation rate has  $T/\gamma_v$ -linear dependence where  $\gamma_v$  is the inverse of the reduced magnetic susceptibility. The  $T$ -linear coefficient of the specific heat has  $\ln(1+1/\gamma_{0v})$  ( $v = \parallel$  or  $\perp$ ) where  $\gamma_{0v}$  is the inverse of the reduced magnetic susceptibility at the zero temperature.

### Introduction

The magnetic properties of nearly ferromagnetic metals have intrigued the interest of many experimental and theoretical researchers [1-14]. Recently, the anisotropic spin fluctuations were investigated in nearly antiferromagnetic metals beyond the random phase approximation [15, 16]. However, in the nearly ferromagnetic metals, the influence of the anisotropic spin fluctuations has not been resolved. Therefore, the self-consistent renormalization theory of anisotropic spin fluctuations in the nearly ferromagnetic metals is constructed beyond the random phase approximation in this paper. The inverse of the magnetic susceptibility is investigated. The nuclear magnetic relaxation rate is studied. The  $T$ -linear coefficient of the specific heat is examined. Throughout this paper, we use units of energy, such that

$\hbar = 1$ ,  $k_B = 1$ , and  $g\mu_B=1$  where  $g$ -factor of the conduction electron, unless explicitly stated. We assume that the  $c$ -axis is the axis of easy magnetization.

This paper is organized as follows: the formulation will be provided in section 2. The numerical results will be supplied in section 3. The conclusions will be given in section 4.

## The Inverse of the Magnetic Susceptibility with the SCR Theory

Let's begin the non-interacting dynamical susceptibility. By using Moriya's expression [14] based on the single band Hubbard model, the non-interacting dynamical susceptibility  $\chi_{0v}(q, \omega)$  as follows:

$$\chi_{0v}(q, \omega) = \chi_{0v}(0) \left(1 - Aq^2 + ic \frac{\omega}{q}\right), (v = || \text{ or } \perp) \quad (1)$$

$$s_{Lv}^2(T) = \frac{1}{\pi} \sum_q \int_0^\infty d\omega \frac{1}{e^{\omega/T} - 1} \text{Im} \chi_v(q, \omega) \quad (2)$$

$$\text{Im} \chi_v(q, \omega) = \frac{2\pi T_0}{\alpha_v T_{A_v}} \frac{\omega}{\mu_{lv}^2 + \omega^2} \quad (3)$$

with

$$\mu_{lv} = 2\pi T_0 (1/2\alpha_v T_{A_v} \chi_v(0)) + (q/q_B)^2, \quad (4)$$

$$T_{A_v} = Aq_B^2 / 2 \quad (5)$$

$\chi_{0v}(0)$  is the non-interacting magnetic susceptibility.  $q_B$  is the magnitude of the zone boundary wave vector. From Eq. (2),  $s_{Lv}^2(T)$  is

$$s_{Lv}^2(T) = \frac{3T_0}{\alpha_v T_{A_v}} \int_0^1 dx x^3 \left( \ln \mu_v - \frac{1}{2\mu_v} - \psi(\mu_v) \right) \quad (6)$$

where  $\psi(\mu_v)$  is the digamma function,

$$y_v = \frac{1}{2\alpha_v T_{A_v} \chi_v(0)}, \quad (7)$$

$$t = T/T_0, \mu_v = x(x^2 + y_v) / t \quad (8)$$

where  $y_v$  ( $v = \parallel$  or  $\perp$ ) is the inverse of the reduced magnetic susceptibility.  $y_{\parallel}$ ,  $y_{\perp}$  are parallel to the c-axis and perpendicular to c-axis, respectively. By following Ref. [6]

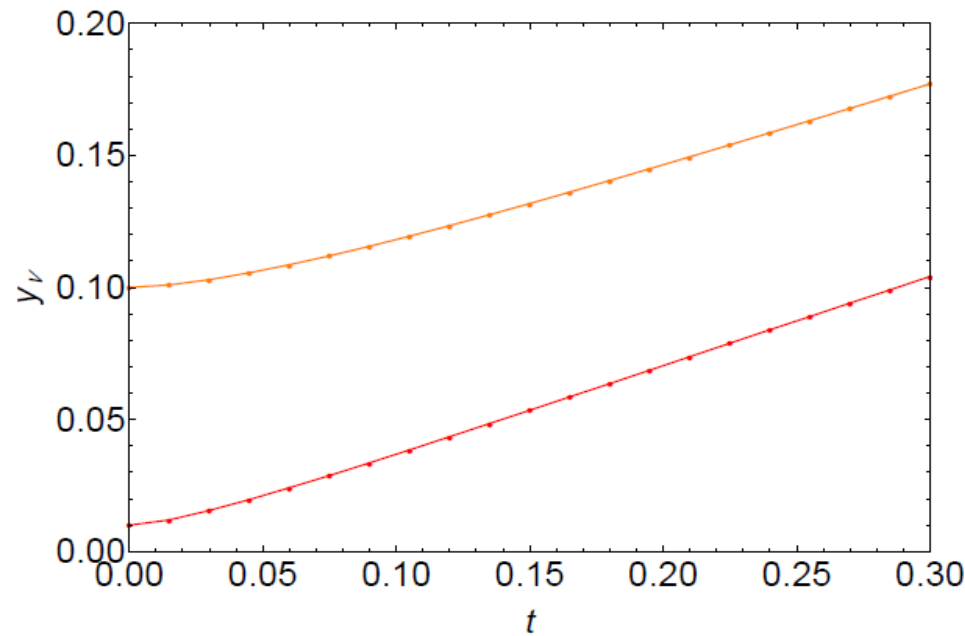
$$1/x_{\parallel}(T) = 1/x_{\parallel}(0,0) + 3F_{\parallel}S_{L\parallel}^2(T) + 2F_{\perp}S_{L\perp}^2(T) \quad (9)$$

$$1/x_{\perp}(Q,T) = 1/x_{\perp}(0,0) + F_{\parallel}S_{L\parallel}^2(T) + 4F_{\perp}S_{L\perp}^2(T) \quad (10)$$

From Eqs. (9) and (10), the equations of the inverse of the reduced magnetic susceptibility are obtained.

$$y_{\parallel} = y_{0\parallel} + (3/2)y_{1\parallel} \int_0^1 dx x^3 [\ln \mu_{\parallel} - 1/(2\mu_{\parallel}) - \psi(\mu_{\parallel})] \\ + (2/2)y_{1\perp} \int_0^1 dx x^3 [\ln \mu_{\perp} - 1/(2\mu_{\perp}) - \psi(\mu_{\perp})] \quad (11)$$

**Figure 1:** The temperature dependence of the inverse of the reduced magnetic susceptibility  $y_v$  ( $v = \parallel$ , or  $\perp$ ) when  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 3$ , and  $y_{1\perp} = y_{2\perp} = 1$ , respectively.



$$y_{\perp} = y_{0\perp} + (1/2)y_{2\parallel} \int_0^1 dx x^3 [\ln \mu_{\parallel} - 1/(2\mu_{\parallel}) - \psi(\mu_{\parallel})] \\ + 4y_{2\perp} \int_0^1 dx x^3 [\ln \mu_{\perp} - 1/(2\mu_{\perp}) - \psi(\mu_{\perp})] \quad (12)$$

$$y_{0v} = \frac{1}{2\alpha_v T_{Av} x_v(Q,0)}, \quad (13)$$

$$y_{1\parallel} = \frac{3F_{\parallel}T_0}{(\alpha_{\parallel}T_{A\parallel})^2}, \quad (14)$$

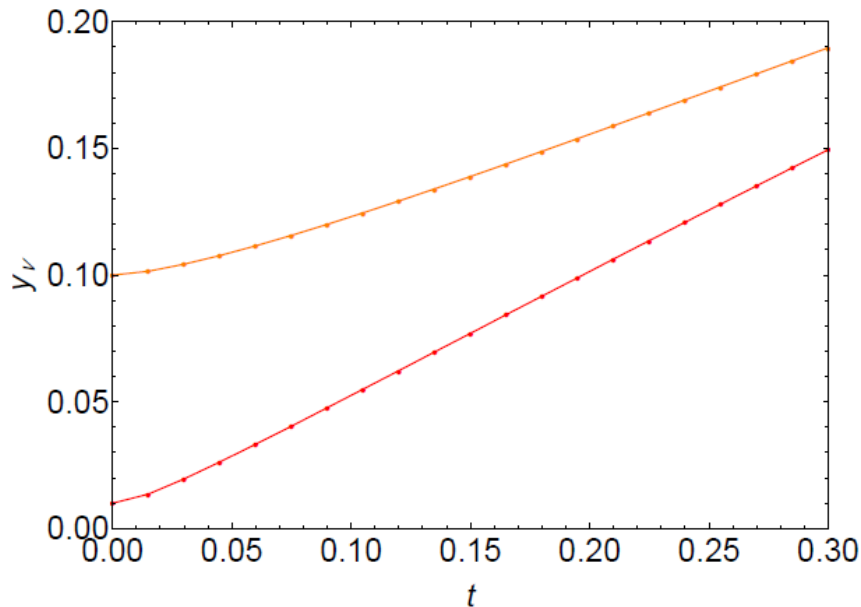
$$y_{1\perp} = \frac{3F_{\perp}T_0}{\alpha_{\parallel}\alpha_{\perp}T_{A\parallel}T_{A\perp}}, \quad (15)$$

$$y_{2\parallel} = \frac{F_{\parallel}T_0}{\alpha_{\parallel}\alpha_{\perp}T_{A\parallel}T_{A\perp}}, \quad (16)$$

$$y_{2\perp} = \frac{3F_{\perp}T_0}{2(\alpha_{\perp}T_{A\perp})^2}, \quad (17)$$

Fig.1 shows the temperature dependence of  $y_v$  ( $v = \parallel, or \perp$ ) with  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 3$ , and  $y_{1\perp} = y_{2\perp} = 1$ . Fig.2 shows the temperature dependence of  $y_v$  ( $v = \parallel, or \perp$ ) with  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 6$  and  $y_{1\perp} = y_{2\perp} = 1$ .

**Figure 2:** The temperature dependence of the inverse of the reduced magnetic susceptibility  $y_v$  ( $v = \parallel, or \perp$ ) when  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 6$ ,  $y_{1\perp} = y_{2\perp} = 1$ , respectively.



The inverse of the reduced magnetic susceptibility has  $T$ -linear dependence from Fig.1 and Fig.2. At low temperatures  $t \ll 1$ , we use the following asymptotic expansion of the digamma function in the integrand of Eqs. (11) and (12).

$$\ln \mu_v - 1/2 \mu_v - \psi(\mu_v) \approx \frac{1}{12\mu_v^2} + \dots \quad (18)$$

At low temperatures the inverse of the magnetic susceptibility has  $T^2$ -linear dependence. In elevated temperatures, it has  $T$ -linear dependence.

### The Nuclear Magnetic Relaxation Rate

The nuclear magnetic relaxation rate is studied by using the dynamical susceptibility in the nearly ferromagnetic metals. It is obtained:

$$\frac{1}{T_{1\nu}T} = \frac{1}{N_0} \gamma_n^2 A_{hf}^2 \sum_q \frac{\text{Im} x_\nu(\mathbf{q}, \omega_0)}{\omega_0} \quad (19)$$

where  $T_{1\nu}$  ( $\nu = \parallel, \text{or } \perp$ ) is a nuclear magnetic relaxation time,  $A_{hf}$  is the hyperfine coupling constant.  $\gamma_n$  is the nuclear gyromagnetic ratio, and  $N_0$  is the number of the magnetic atom. The nuclear magnetic relaxation rate in the nearly ferromagnetic metal is where  $g$  is the  $g$ -factor of the conduction electron, and  $\mu_B$  is the Bohr's magneton.

$$\frac{1}{T_{1\nu}} = (g \mu_B)^2 (\gamma_n A_{hf})^2 t \frac{3}{2\pi\alpha_\nu T_{Av}} \left[ \frac{1}{y_\nu} - \frac{1}{1+y_\nu} \right] (\nu = \parallel \text{ or } \perp) \quad (20)$$

**Figure 3:** The temperature dependence of  $\frac{2\pi\alpha_\nu T_{Av}}{T_{1\nu} (\gamma_n A_{hf})^2 (g \mu_B)^2}$  ( $\nu = \parallel \text{ or } \perp$ ) when  $y_{0\parallel} = 0.01$  (the red line),

$y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 3$ ,  $y_{1\perp} = y_{2\perp} = 1$ , respectively.

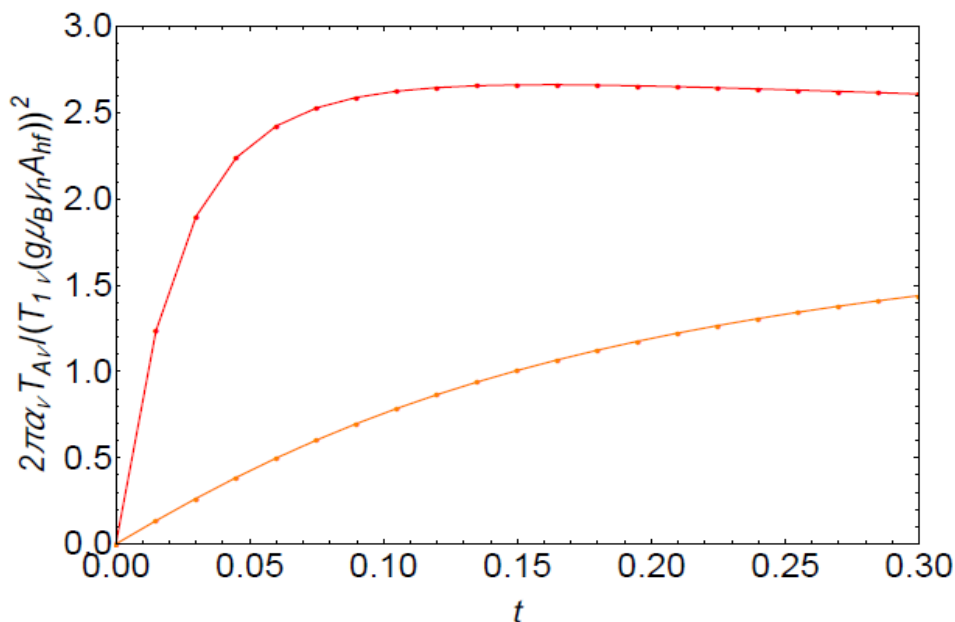


Fig. 3 shows the temperature dependence of the nuclear magnetic relaxation rate with  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 3$  and  $y_{1\perp} = y_{2\perp} = 1$ . Fig. 4 shows the temperature dependence of the nuclear magnetic relaxation rate with  $y_{0\parallel} = 0.01$  (the red line),  $y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 6$  and  $y_{1\perp} = y_{2\perp} = 1$ . From Eq. (20), the nuclear magnetic relaxation rate has  $t/y_v$ -linear dependence because of  $y_v \ll 1$  in nearly ferromagnetic metals. In contrast to nearly ferromagnetic metals, the nuclear magnetic relaxation rate has  $t(1 - \sqrt{y_{sv}} \pi/2)$ -linear dependence in nearly antiferromagnetic metals where  $y_{sv}$  is the inverse of the staggered magnetic susceptibility [16]. .

### The T-linear Coefficient of the Specific Heat

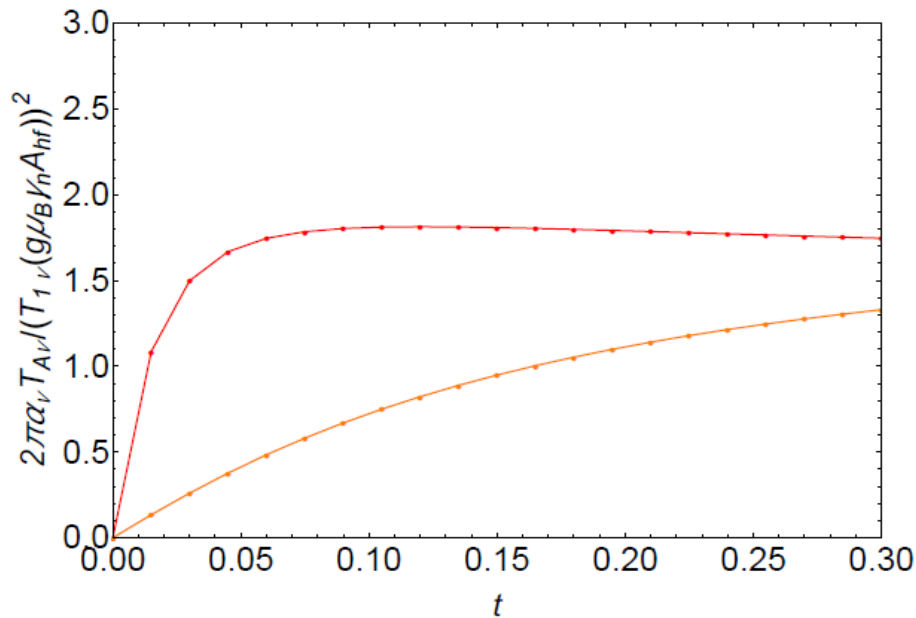
The free energy of spin fluctuations is obtained as follows [5]:

$$F_{sf} = \int_0^{\omega_c} d\omega f(\omega) \sum_{q,v} \frac{1}{\pi \Gamma_{qv}^2 + \omega^2} \quad (21)$$

$$f(\omega) = \frac{\omega}{2} + T \ln(1 - e^{-\omega/T}) \quad (22)$$

**Figure 4:** The temperature dependence of  $\frac{2\pi\alpha_v T_{Av}}{T_{1v} (\gamma_n A_{hf})^2 (g \mu_B)^2}$  ( $v = \parallel$  or  $\perp$ ) when  $y_{0\parallel} = 0.01$  (the red line),

$y_{0\perp} = 0.1$  (the orange line),  $y_{1\parallel} = y_{2\parallel} = 6$ ,  $y_{1\perp} = y_{2\perp} = 1$ , respectively.



where  $\Gamma_{qv}$  is the damping constant of spin fluctuations ( $v = \parallel$  or  $\perp$ ). The specific heat of spin fluctuations is

$$\frac{c_m}{T} = -\frac{\partial^2 F_{sf}}{\partial T^2} \quad (23)$$

$$\gamma_m = \frac{N_0}{4T_0} \ln\left(1 + \frac{1}{y_{0\parallel}}\right) + \frac{N_0}{2T_0} \ln\left(1 + \frac{1}{y_{0\perp}}\right). \quad (24)$$

From Eq. (24),  $\gamma_m$  increases when  $y_{0v}$  decreases. In contrast to nearly ferromagnetic metals,  $\gamma_m$  increases in nearly antiferromagnetic metals when  $y_{s0}$  increases where  $y_{s0}$  is the inverse of the reduced staggered magnetic susceptibility at the zero temperature.

## Conclusions

We have made the self-consistent renormalization theory of anisotropic spin fluctuations in three dimensional nearly ferromagnetic metals beyond the random phase approximation. We have investigated the temperature dependence of the inverse of the magnetic susceptibility, nuclear magnetic relaxation rate, and the  $T$ -linear coefficient of the specific heat in nearly ferromagnetic metals. We have found that the temperature dependence of the inverse of the magnetic susceptibility has  $T^2$ -linear behavior at low temperatures. With increasing temperatures, it has  $T$ -linear dependence. The nuclear magnetic relaxation rate has  $t/y_v$ -linear dependence. The anisotropy appears in the inverse of the magnetic susceptibility and the nuclear magnetic relaxation rate by anisotropic spin fluctuations.

## Acknowledgments

This work is supported by the Kindai University Technical College grants. The author would like to thank Y. Takahashi, D. Legut, and S. Khmelevskiy for fruitful discussions. He would like to also thank Y. Tokunaga, Y. Haga, H. von Lohneysen, M. Brando, F. Steglich, C. Geibel, J. Flouquet, A. de Visser, S. Murayama, E. Bauer, M. Grosche, K. Gofryk, P. Canfield, F. Honda, K. Ishida and V. Sechovsky for stimulating conversations.

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