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Wave Scattering by Many Small Impedance Particles and Applications

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Abstract

Formulas are derived for solutions of many-body wave scattering problem by small impedance particles embedded in a homogeneous medium. The limiting case is considered, when the size a of small particles tends to zero while their number tends to infinity at a suitable rate. The basic physical assumption is $a \ll d \ll \lambda$, where d is the minimal distance between neighboring particles, λ is the wavelength, and the particles can be impedance balls $B\ (xm,\ a)$ with centers xm located on a grid. Equations for the limiting effective (self-consistent) field in the medium are derived. It is proved that one can create material with a desired refraction coefficient by embedding in a free space many small balls of radius a with prescribed boundary impedances. The small balls can be centered at the points located on a grid. A recipe for creating materials with a desired refraction coefficient is formulated. It is proved that materials with a desired radiation pattern, for example, wave-focusing materials, can be created.

Introduction

There is a large literature on wave scattering by small bodies, starting from Rayleigh's work (1871), [1, 2, 36]. For the problem of wave scattering by one body an analytical solution was found only for the bodies of special shapes, for example, for balls and ellipsoids. If the scatterer is small then the scattered field can be calculated analytically for bodies of arbitrary shapes, see [5], where this theory is presented. The many-body wave scattering problem was discussed in the literature mostly numerically, if the number of scatterers is small, or under the assumption that the influence of the waves, scattered by other particles on a particular particle is negligible (see [3], where one finds a large bibliography, 1386 entries). This corresponds to the case when *the distance* d *between neighboring particles is much larger than the wavelength* λ*, and the characteristic size* a *of a small body (particle) is much smaller than* λ*.* Theoretically and practically the assumptions $a \ll \lambda$, $d >> \lambda$ are the simplest and they allow to neglect multiple scattering. By $k = \frac{2\pi}{\lambda}$ the wave number is denoted. In contrast, in our theory the basic assumption is $a \ll d \ll \lambda$, and *the multiple scattering is of basic importance*. We give references to our papers and monographs in which the theory of wave scattering by small bodies of arbitrary shapes was developed under the assumption $a \ll d \ll \lambda$, [4–34]. The novelty of the results in this paper is in the location of the small bodies: *they are placed on a grid.* This may be of practical interest. In [35] for the first time the scattering problem for 10 billions small particles is solved numerically and numerical results are presented. This paper is a presentation of the new results under simplifying assumptions: the small particles $D_m = B(x_m, a)$, $1 \le m \le M$, are impedance balls with prescribed boundary impedances ζ_m; the centers x_m of the balls are placed on a grid and are embedded in a homogeneous space in a bounded domain D, for example, in a box. The basic results of this paper consist of:

1. Solution to *many-body wave scattering problem* by small impedance particles, embedded in a homogeneous medium, under the assumptions $a \ll d \ll \lambda$, where d is the minimal distance between neighboring particles and λ is the wavelength in this medium.

2. Derivation of the equations for the limiting effective (self-consistent) field in this medium, in which many small impedance particles are embedded, when $a \to 0$ and the number $M = M(a)$ of the small particles tends to infinity at an appropriate rate.

3. Derivation of linear algebraic systems (LAS) for solving many-body wave scattering problems. These systems are not obtained by a discretization of boundary integral equations, and they give an efficient numerical method for solving many- body wave scattering problems in the case of small scatterers under the assumption $a \ll d \ll \lambda$.

4. Formulation of a recipe for creating materials with a desired refraction coefficient.
5. Formulation of a method for creating materials with a desired radiation pattern.

Our methods give powerful numerical methods for solving many-body wave scattering problems in the case when the scatterers are small (see [31]).

Let us formulate the wave scattering problems we deal with. Let D be a bounded domain in $\mathbb{R}^3$ with a sufficiently smooth boundary. The scattering problem consists of finding the solution to the problem:

$$(\nabla^2 + k^2)u = 0 \text{ in } G' := \mathbb{R}^3 \setminus G, \qquad G := \cup_{m=1}^{M} D_m, \qquad k = \text{const} > 0, \tag{1}$$

where $D_m = B(x_m, a)$ is an impedance ball, centered at x_m and of small radius a,

$$u = u_0 + v, \qquad u_0 = e^{ik\alpha\cdot x}, \qquad \alpha \in S^2, \tag{2}$$

S^2 is the unit sphere in $\mathbb{R}^3$, u_0 is the incident field, v is the scattered field satisfying the radiation condition

$$v_r - ikv = o\left(\frac{1}{r}\right), \qquad r := |x| \to \infty, \qquad v_r := \frac{\partial v}{\partial r}, \tag{3}$$

and u satisfies the impedance boundary condition (bc) on the boundary of G,

$$u_N - \zeta_m u = 0, \qquad \text{on } S_m, \qquad \text{Im}\zeta_m \le 0, \tag{4}$$

where ζ_m is a constant, N is the unit normal to $S := \cup_{m=1}^{M} S_m$, pointing out of $G := \cup_{m=1}^{M} D_m$, and S_m is the surface of $D_m = B(x_m, a)$.

By refraction coefficient $n(x)$ the coefficient in the equation

$$(\nabla^2 + k^2 n^2(x))u = (\nabla^2 + k^2 - q(x))u = 0 \tag{5}$$

is understood, where $q(x) := k^2(n^2(x) - 1)$.

Let

$$g(x, y) = \frac{e^{ik|x-y|}}{4\pi|x-y|}.$$

Then $(\nabla^2 + k^2)g(x, y) = -\delta(x - y)$, where $\delta(x)$ is the delta function.

Let us distribute small impedance particles $D_m = B(x_m, a)$ in D so that

$$\mathbb{N}(\Delta) = a^{\kappa-2}|\Delta|[1 + o(1)], \qquad a \to 0, \tag{6}$$

where $\Delta \subset D$ is an arbitrary connected open subset of D, $|\Delta|$ is its volume, $\kappa \in (0, 1)$ is a number the experimenter may choose arbitrarily and $\mathbb{N}(\Delta)$ is the number of particles in Δ. Throughout this paper the important assumptions $a \ll d \ll \lambda$ and (6) are satisfied. As $a \to 0$ the number of small particles $\mathbb{N}(\Delta)$ in (6) tends to infinity since $\kappa - 2 < 0$.

The boundary impedances ζ_m are chosen by the formula

$$\zeta_m = a^{-\kappa} h(x_m), \tag{7}$$

where $h(x)$ is a continuous function in D, $\text{Im}h \le 0$.

It will be clear from Section 3 that the function $h(x)$ can be determined by choosing a suitable boundary impedance $\zeta(x)$. When $a \to 0$, the ζ_m and $h(x_m)$ can be considered as continuous functions $\zeta(x)$ and $h(x)$.

The many-body scattering problem (1)–(4) has a solution and this solution is unique, see [31] In Section 2 a method for solving this problem is given. In Section 3 a recipe for creating materials with a desired refraction coefficient is given. In Section 4 a recipe for creating materials with a desired radiation pattern is given.

Solution of Many-Body Scattering Problem

We look for the solution of the form

$$u = u_0 + \sum_{m=1}^{M} \int_{S_m} g(x,s)\sigma_m(s)ds = \sum_{m=1}^{M} g(x,x_m)Q_m + J, \qquad (8)$$

where $\sigma_m(s)$ are unknown, $Q_m := \int_S \sigma_m(s)ds$. One may think about σ_m as of charge densities on S_m and of Q_m as of total charge on the surface S_m. We prove that is negligible compared to as $a \to 0$.

$$J := \sum_{m=1}^{M} \int_{S_m} [g(x,s) - g(x,x_m)]\sigma_m(s)ds$$

$$I := \sum_{m=1}^{M} g(x,x_m)Q_m, \qquad J \ll I$$

Let us prove this claim. First, we need the following lemma.

LEMMA 1. *One has:*

$$Q_m = -4\pi a^2 \zeta_m u_m = -4\pi a^{2-\kappa} h_m u_m, \qquad h_m := h(x_m), \qquad u_m := u(x_m). \qquad (9)$$

Proof: Let us define *the effective field acting on the m-th body,*

$$u_e := u_e^m := u - \int_{S_m} g(x,s)\sigma_m(s)ds.$$

If a is small, then $u(x) \sim u_e(x)$ for any x such that $|x - x_m| \geq d$. Let us use the exact boudary condition (4) for u_e and the known formula for the normal derivative of the single layer potential to get

$$u_{eN} + (A\sigma_m - \sigma_m)/2 - \zeta_m u_{em} - \zeta_m \int_{S_m} g(x,s)\sigma_m(s)ds = 0. \qquad (10)$$

Here $A\sigma := \int_{S_m} g_{N_t}(t,s)\sigma_m(s)ds$, $t \in S_m$. Let us integrate (10) over S_m and keep the main term as $a \to 0$. One knows that $\int_{S_m}(A\sigma - \sigma)/2dt = -Q_m$. Furthermore, $\int_{S_m} g(t,s)ds = a$, as one can check by a simple calculation using the fact that S_m is a sphere of radius a. This allows one to conclude that

$$\zeta_m \int_{S_m} ds\sigma_m(s) \int_{S_m} g(t,s)dt = h_m a^{1-\kappa} Q_m, \qquad \zeta_m \int_{S_m} u_e ds = -4\pi a^{2-\kappa} h_m u_{em}$$

and $\int_{S_m} u_{eN} ds = O(a^2)$ as $a \to 0$. From the above estimates the conclusion of Lemma 1 follows. □

Let us now check our claim $J \ll I$ as $a \to 0$. One has

$$g(x, x_m)Q_m = O(a^{2-\kappa}d^{-1})$$

for $|x - x_m| > d$, $a \to 0$. On the other hand, one derives

$$\left|\int_{S_m}[g(t,s) - g(x,x_m)]\sigma_m(s)ds\right| \le O(ad^{-2}a^{2-\kappa}) = O\left(\frac{a}{d}\right)O(a^{2-\kappa}d^{-1}).$$

This estimate justifies our claim since $a \ll d$. It follows that asymptotically, as $a \to 0$, one has for $|x - x_m| \ge a$. Note that $M = O(a^{\kappa-2})$. Formula (11) allows one to calculate $u(x)$ at any point x, if the numbers u_m, $1 \le m \le M$, are known. One can use the following linear algebraic system (LAS) for finding u_m.

$$u \sim u_0 + \sum_{m=1}^{M} g(x, x_m)Q_m \sim u_0 - 4\pi a^{2-\kappa}\sum_{m=1}^{M} g(x, x_m)h_m u_m,$$

$$u_j = u_{0j} - 4\pi a^{2-\kappa}\sum_{m\ne j}^{M} g(x_j, x_m)h_m u_m, \qquad 1 \le j \le M.$$

The order $M = O(a^{\kappa-2})$ of this system is large if a is small. One can reduce this order: consider a covering of D by nonintersecting small cubes Δ_p, $1 \le p \le P$, such that $d \ll \text{diam}(\Delta_p) \ll \lambda$, $u_m \sim u_p$, $h_m \sim h_p$ for all $x_m \in \Delta_p$. Then formula (12) can be written as by formula (6). As $a \to 0$, $\text{diam}(\Delta_p) \to 0$ and formula (13) yields in the limit the integral equation for u.

$$u_q = u_{0q} - 4\pi a^{2-\kappa}\sum_{p\ne q}^{P} g(x_q, x_p)h_p u_p \sum_{x_m \in \Delta_p} 1 = u_{0q} - 4\pi \sum_{p\ne q}^{P} g(x_q, x_p)h_p u_p|\Delta_p|,$$

where

$$a^{2-\kappa}\sum_{x_m \in \Delta_p} 1 = |\Delta_p|$$

$$u(x) = u_0(x) - 4\pi \int_D g(x, y)h(y)u(y)dy.$$

LEMMA 2. Eq. (14) has a solution, this solution is unique and it is a limiting value of the solution to the scattering problem (1)– (4).

Proof: Apply the operator $\nabla^2 + k^2$ to equation (14) and get $(\nabla^2 + k^2)u = 4\pi h(x)u(x)$.

This is a Schrödinger equation with potential $q(x) := 4\pi h(x)$; equations (2)–(3) hold. We assumed Im$h \leq 0$. Therefore (15) has at most one solution. It is a Fredholm-type equation, so it has a solution. Lemma 2 is proved. □

It follows from Lemma 2 that the LAS (13) for u_p is solvable and its solution is unique. Let us write Eq. (15) as

$$\nabla^2 u + k^2 n^2(x) u = 0, \quad n^2(x) := 1 - 4\pi k^{-2} h(x)$$

Conclusion

Embedding small impedance balls $B(xm, a)$ in D results in creating in D a new material with the refraction coefficient $n(x) = (1 - 4\pi k{-}2\ h(x))1/2$.

If one wants to have a material with the refraction coefficient $n(x)$, then one chooses by (17) the function $h(x)$. If $h(x)$ is chosen, then one knows the boundary impedance $\zeta(x)$ which generates the desired $h(x)$. The practical problem is to prepare small particles with the desired boundary impedance.

Recipe for Creating Materials with A Desired Refraction Coefficient

Let us formulate a recipe for creating materials with a desired refraction coefficient. Formula (17) shows that if $h(x)$ is chosen properly, then any $n(x)$ can be obtained in D.

Recipe for creating materials with a desired refraction coefficient:

a) Calculate by formula (17) the function $h(x)$;

b) Distribute small impedance balls in the domain D by the distribution law (6).

The boundary impedances of these balls are defined by the function $h(x)$.

Theorem 1. The refraction coefficient of the resulting medium tends to the desired coefficient $n(x)$ as $a \to 0$.

Let us show that a practically negative refraction coefficient $n(x)$ can be obtained by the above recipe. Denote $b := 4\pi k{-}2 > 0$ and write (17) as $n(x) = (1{-}bh(x))1/2 = |1{-}bh(x)|1/2\ e\ \phi/2$, where ϕ is the argument of $1 - bh(x)$. Since the operator in (14) is Fredholm, it remains Fredholm under small perturbations. Therefore, one can take $h - i\epsilon$, where $\epsilon > 0$ is sufficiently small and equation (14) will still have a unique solution.

By choosing h so that Re $(1 - bh) > 0$ and Im$(1 - bh) < 0$ and small, one gets the argument $\phi = 2\pi - \delta$, where $\delta > 0$ is arbitrarily small if ϵ is sufficiently small. Then $n(x)$ will be nearly negative: its argument will be $\pi - \delta/2$.

Creating Materials with A Desired Radiation Pattern

Let us define what we mean by radiation pattern. Consider the scattering problem for Eq. (15)

$$\nabla^2 u + k^2 u - q(x) u = 0, \qquad u = e^{ik\alpha\cdot x} + v,$$

where v satisfies the radiation condition. Assume that $k > 0$ and $\alpha \in S^2$ are fixed. Then the scattering amplitude $A(\beta, \alpha, k) = A(\beta)$, where the dependence on k, α is dropped since k and α are fixed. The formula for the scattering amplitude is known, see, e.g. [34],

$$A(\beta) := A_q(\beta) = -\frac{1}{4\pi}\int e^{-ik\beta\cdot y}q(y)u(y)dy.$$

We call $A(\beta)$ the radiation pattern. Consider an inverse problem (IP): Given an arbitrary $f(\beta) \in L^2(S^2)$ and an arbitrary small $\epsilon > 0$, can one find a $q \in L^2(D)$ such that

$$\|f(\beta) - A_q(\beta)\|_{L^2(S^2)} < \epsilon.$$

Theorem 2. For any $f(\beta) \in L^2(S^2)$ and an arbitrary small $\epsilon > 0$ there is a $q \in L^2(D)$ such that (20) holds.

Since small perturbations of q result in small perturbations of $A(\beta)$, there are infinitely many potentials q for which inequality (20) holds.

The conclusion of Theorem 2 follows from Lemmas 3 and 4.

Lemma 3. The set $\{\int_D e^{-ik\beta\cdot x} h(x)dx\}_{\forall h\in L^2(D)}$ is dense in $L^2(S^2)$.

Corollary 1. Given $f \in L^2(S^2)$ and $\epsilon > 0$, one can find $h \in L^2(D)$ such that

$$\|f(\beta) + \frac{1}{4\pi}\int_D e^{-ik\beta\cdot x}h(x)dx\| < \epsilon.$$

Lemma 4. The set $\{q(x)u(x, \alpha)\}_{\forall q\in L^2(D)}$ is dense in $L^2(D)$.

Corollary 2. Given $h \in L^2(D)$ and $\epsilon > 0$, one can find $q \in L^2(D)$ such that

$$\|h(x) - q(x)u(x,\alpha)\|_{L^2(D)} < \epsilon.$$

Since the scattering amplitude depends continuously on h, the inverse problem IP is solved by Lemmas 3 and 4.

$$A(\beta) = -\frac{1}{4\pi}\int_D e^{-ik\beta\cdot x}h(x)dx$$

Proof of Lemma 3: Assume the contrary. Then $\exists\ \psi \in L^2(S^2)$ such that

$$0 = \int_{S^2} d\beta\psi(\beta) \int_D e^{-ik\beta\cdot x} h(x)dx \qquad \forall h \in L^2(D).$$

Thus,

$$\int_{S^2} d\beta\psi(\beta)e^{-ik\beta\cdot x} = 0 \qquad \forall x \in \mathbb{R}^3.$$

Therefore,

$$\int_0^\infty d\lambda\lambda^2 \int_{S^2} d\beta e^{-i\lambda\beta\cdot x}\psi(\beta)\frac{\delta(\lambda-k)}{k^2} = 0 \qquad \forall x \in \mathbb{R}^3.$$

By the injectivity of the Fourier transform, one gets

$$\psi(\beta)\frac{\delta(\lambda-k)}{k^2} = 0.$$

Therefore, $\psi(\beta) = 0$. Lemma 3 is proved. □

Proof of Lemma 4: Given $h \in L^2(D)$, define

$$u := u_0 - \int_D g(x,y)h(y)dy, \qquad g := \frac{e^{ik|x-y|}}{4\pi|x-y|}, \tag{21}$$

$$q(x) := \frac{h(x)}{u(x)}. \tag{22}$$

If $q \in L\ 2\ (D)$, then this q solves the problem, and u, defined in (21), is the scattering solution and If q is not in $L\ 2\ (D)$, then the null set $N := \{x : x \in D, u(x) = 0\}$ is non-void.

$$u = u_0 - \int_D g(x,y)q(y)u(y)dy,$$

$$A(\beta) = -\frac{1}{4\pi}\int_D e^{-ik\beta\cdot y}h(y)dy.$$

Let

$$N_\delta := \{x : \ |u(x)| < \delta, x \in D\}, \qquad D_\delta := D \setminus N_\delta.$$

Claim 1. $\exists h_\delta = \begin{cases} h, \textit{in}\ D_\delta, \\ 0, \textit{in}\ N_\delta, \end{cases}$ *such that* $\|h_\delta - h\|_{L^2(D)} < c\epsilon,$

$$q_\delta := \begin{cases} \frac{h_\delta}{u_\delta}, \textit{in}\ D_\delta, \\ 0, \ \ \textit{in}\ N_\delta, \end{cases} \ q_\delta \in L^\infty(D), \qquad u_\delta := u_0 - \int_D g h_\delta dy.$$

Proof of Claim 1*:* The set N is, generically, a line $l = \{x : u_1(x) = 0,\ u_2(x) = 0\}$, where $u_1 = \Re u$ and $u_2 = \Im u$. Consider a tubular neighborhood of this line, $\rho(x, l) \le \delta$. Let the origin O be chosen on l, s_3 be the Cartesian coordinate along the tangent to l, and $s_1 = u_1$, $s_2 = u_2$ are coordinates in the plane orthogonal to l, s_j-axis is directed along $\nabla u_j|_l$, $j = 1, 2$.

The Jacobian $\mathcal{J}$ of the transformation $(x_1, x_2, x_3) \mapsto (s_1, s_2, s_3)$ is nonsingular, $|\mathcal{J}| + |\mathcal{J}^{-1}| \le c$, because ∇u_1 and ∇u_2 are linearly independent. Define

$$h_\delta := \begin{cases} h, \text{in } D_\delta, \\ 0, \text{in } N_\delta, \end{cases} \qquad u_\delta := u_0 - \int_D g(x,y)h_\delta(y)dy, \qquad q_\delta := \begin{cases} \frac{h_\delta}{u_\delta}, \text{in } D_\delta, \\ 0, \quad \text{in } N_\delta. \end{cases}$$

One has $u_\delta = u_0 - \int_D ghdy + \int_D g(x,y)(h - h_\delta)dy$,

$$|u_\delta(x)| \ge |u(x)| - c\int_{N_\delta} \frac{dy}{4\pi|x-y|} \ge \delta - I(\delta), \qquad x \in D_\delta, \qquad c = \max_{x\in N_\delta} |h(x)|.$$

If one proves that $I(\delta) = o(\delta)$, $\delta \to 0$, $\forall x \in D_\delta$ then $q_\delta \in L^\infty(D)$, and Claim 1 is proved. □

Claim 2.

$$I(\delta) = O(\delta^2|\ln(\delta)|), \qquad \delta \to 0.$$

Proof of Claim 2:

$$\int_{N_\delta} \frac{dy}{|x-y|} \le \int_{N_\delta} \frac{dy}{|y|} = c_1 \int_0^{c_2\delta} \rho \int_0^1 \frac{ds_3}{\sqrt{\rho^2 + s_3^2}} d\rho$$
$$= c_1 \int_0^{c_2\delta} d\rho\rho \ln(s_3 + \sqrt{\rho^2 + s_3^2})|_0^1 \le c_3 \int_0^{c_2\delta} \rho \ln\left(\frac{1}{\rho}\right) d\rho$$
$$\le O(\delta^2|\ln(\delta)|).$$

The condition $|\nabla u\ j\ |l \ge c > 0$, $j = 1, 2$, implies that a tubular neighborhood of the line l, $N\delta = \{x : \sqrt{}\ |u1|\ 2 + |u2|\ 2 \le \delta\}$, is included in a region $\{x : |x| \le c2\delta\}$ and includes a region $\{x : |x| \le c\ 0\ 2\ \delta\}$. This follows from the estimates

$$c_2'\rho \le |u(x)| = |\nabla u(\xi) \cdot (x - \xi)| \le c_2\rho.$$

Here $\xi \in l$, x is a point on a plane passing through ξ and orthogonal to l, $\rho = |x - \xi|$, and $\delta > 0$ is sufficiently small, so that the terms of order $\rho\ 2$ are negligible, $c2 = \max\xi \in l\ |\nabla u(\xi)|$, $c_2 = \min\xi \in l\ |\nabla u(\xi)|$.

Claim 2, and, therefore, Lemma 4 are proved.

Therefore, Theorem 2 is proved.

Let us describe a numerical method for calculation of h given $f(\beta)$ and $\epsilon > 0$. Let $\{\phi_j\}$ be a basis in $L^2(D)$, $h_n = \sum_{j=1}^n c_j^{(n)} \phi_j$, $\psi_j(\beta) := -\frac{1}{4\pi} \int_D e^{-ik\beta \cdot x} \phi_j(x) dx$. Consider the problem

$$\|f(\beta) - \sum_{j=1}^{n} c_j^{(n)} \psi_j(\beta)\| = \min . \quad (24)$$

A necessary condition for (24) is a linear algebraic system for $c_j^{(n)}$.

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