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Research Article

Self-Consistent Renormalization Theory of Anisotropic Spin Fluctuations in Nearly Antiferromagnetic Metals.

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Abstract

We investigated the temperature dependence of the inverse of the staggered magnetic susceptibility, the nuclear magnetic relaxation rate, and the *T*-linear coefficient of the specific heat in nearly antiferromagnetic metals by using the self-consistent renormalization theory of anisotropic spin fluctuations. At low temperatures, the inverse of the staggered magnetic susceptibility has T^2-linear dependence. In elevated temperatures, the inverse of the staggered magnetic susceptibility has *T*-linear dependence. The nuclear magnetic relaxation rate has *T*-linear dependence at low temperatures. It has $T^{1/2}$-linear dependence in elevated temperatures. The *T*-linear coefficient of the specific heat has $1/\sqrt{y}_{0\nu}$ $(\nu = \| \ or \ \perp)$ where $y_{0\nu}$ is the inverse of the reduced staggered magnetic susceptibility at the zero temperature.

Introduction

The magnetic properties of nearly antiferromagnetic metals have attracted the interest of many experimental and theoretical researchers [1-13]. Recently, the anisotropic spin fluctuations were investigated in quasi-one dimensional nearly antiferromagnetic metal beyond the random phase approximation [14]. However, in the three dimensional nearly antiferromagnetic metals, the influence on the anisotropic spin fluctuations has not been resolved. Therefore, the self-consistent renormalization theory of anisotropic spin fluctuations in the three dimensional nearly

antiferromagnetic metals is constructed beyond the random phase approximation in this paper. The inverse of the staggered magnetic susceptibility is investigated. The nuclear magnetic relaxation rate is studied.

The *T*-linear coefficient of the specific heat is examined. Throughout this paper, we use units of energy, such that $\hbar = 1$, $k_B = 1$, and $g\mu_B = 1$ where g is the g-factor of the conduction electron, unless explicitly stated. We assume that the c-axis is the axis of easy magnetization.

This paper is organized as follows: the formulation will be provided in section 2. The numerical results will be supplied in section 3. The conclusions will be given in section 4.

The inverse of the staggered magnetic susceptibility with the SCR theory

Let's begin the non-interacting dynamical susceptibility. By using Moriya's expression [13] based on the single band Hubbard model, the non-interacting dynamical susceptibility $\chi_{0\nu}(Q+q,\omega)$ as follows:

$$\chi_{0\nu}(Q+q,\omega) = \chi_{0\nu}(Q)(1 - Aq^2 + iC\omega), (\nu = \| \ or \ \perp) \tag{1}$$

Q is the antiferromagnetic staggered wave vector. The square of the local spin amplitude $S^2_{L\nu}(T)$is

$$S^2_{L\nu}(T) = \frac{1}{\pi}\sum_q \int_0^\infty d\omega \frac{1}{e^{\omega/T}-1} \mathrm{Im}\chi_\nu(Q+q,\omega). \tag{2}$$

$\mathrm{Im}\chi_\nu(Q+q,\omega)$ is

$$\mathrm{Im}\chi_\nu(Q+q,\omega) = \frac{2\pi T_0}{\alpha_{s\nu} T_{A_\nu}} \frac{\omega}{u^2_{1\nu} + \omega^2} \tag{3}$$

with

$$u_{1\nu} = 2\pi T_0(1/(2\alpha_{s\nu}\chi_\nu(Q)) + (q/q_B)^2), \tag{4}$$

$$\begin{aligned} T_{A\nu} &= Aq_B^2/2, \\ \Gamma &= A/C, \\ T_0 &= \Gamma q_B^2/(2\pi) \\ \alpha_{s\nu} &= I\chi_{0\nu}(Q), \end{aligned} \tag{5}$$

$\chi_{0\nu}(Q)$is the non-interacting staggered magnetic susceptibility. q_B is the zone boundary wavelength. From Eq. (2), $S^2_{L\nu}(T)$is

$$S_{L\nu}^2(T) = \frac{6T_0}{\alpha_{s\nu}T_{A_\nu}} \int_0^1 dx x^2 (\ln u_\nu - \frac{1}{2u_\nu} - \psi(u_\nu)) \tag{6}$$

where $\psi(u_\nu)$ is the digamma function,

$$y_\nu = \frac{1}{2\alpha_{s\nu}T_{A_\nu}\chi_\nu(Q)}, \tag{7}$$

$$t = T/T_0, u_\nu = (x^2 + y_\nu)/t. \tag{8}$$

By following Ref. [6]

$$1/\chi_\parallel(Q,T) = 1/\chi_\parallel(Q,0) + 3F_{s\parallel}S_{L\parallel}^2(T) + 2F_{s\perp}S_{L\perp}^2(T) \tag{9}$$

$$1/\chi_\perp(Q,T) = 1/\chi_\perp(Q,0) + F_{s\parallel}S_{L\parallel}^2(T) + 4F_{s\perp}S_{L\perp}^2(T) \tag{10}$$

The following inverse of the reduced staggered magnetic susceptibility is introduced. $y_\parallel$, $y_\perp$ are parallel to the c-axis and perpendicular to c-axis, respectively.

$$y_\parallel = y_{0\parallel} + (3/2)y_{1\parallel}\int_0^1 dx x^2[\ln u_\parallel - 1/(2u_\parallel) - \psi(u_\parallel)] + (2/2)y_{1\perp}\int_0^1 dx x^2[\ln u_\perp - 1/(2u_\perp) - \psi(u_\perp)] \tag{11}$$

$$y_\perp = y_{0\perp} + (1/2)y_{2\parallel}\int_0^1 dx x^2[\ln u_\parallel - 1/(2u_\parallel) - \psi(u_\parallel)] + 4y_{2\perp}\int_0^1 dx x^2[\ln u_\perp - 1/(2u_\perp) - \psi(u_\perp)] \tag{12}$$

where,

$$y_{0\nu} = \frac{1}{2\alpha_{s\nu}T_{A\nu}\chi_\nu(Q,0)}, \tag{13}$$

$$y_{1\parallel} = \frac{6F_{s\parallel}T_0}{(\alpha_{s\parallel}T_{A\parallel})^2}, \tag{14}$$

$$y_{1\perp} = \frac{3F_{s\perp}T_0}{\alpha_{s\parallel}\alpha_{s\perp}T_{A\parallel}T_{A\perp}}, \tag{15}$$

$$y_{2\parallel} = \frac{6F_{s\parallel}T_0}{\alpha_{s\parallel}\alpha_{s\perp}T_{A\parallel}T_{A\perp}}, \tag{16}$$

$$y_{2\perp} = \frac{3F_{s\perp}T_0}{(\alpha_{s\perp}T_{A\perp})^2}. \tag{17}$$

Figure 1 shows the temperature dependence of $y_\nu(\nu = \parallel or \perp)$with $y_{0\parallel} = 0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\parallel} = y_{2\parallel} = 3$and $y_{1\perp} = y_{2\perp} = 1$. Fig.2 shows the temperature dependence of $y_\nu(\nu = \parallel or \perp)$ with $y_{0\parallel} = 0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\parallel} = y_{2\parallel} = 1$and $y_{1\perp} = y_{2\perp} = 3$. The inverse of the reduced staggered magnetic susceptibility has *T*-linear dependence from Figure 1 and Figure 2. At low temperatures t << 1, we use the following asymtotic expansion of the digamma function in the integrand of Eqs. (11) and (12).

$$\ln u_\nu - 1/2u_\nu - \psi(u_\nu) \simeq \frac{1}{12u_\nu^2} + \dots. \tag{18}$$

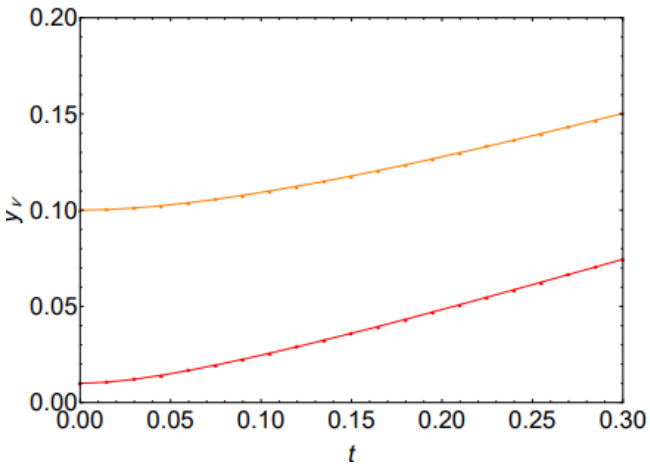
At low temperatures the inverse of the staggered magnetic susceptibility has T^2-linear dependence.

The nuclear magnetic relaxation rate

The nuclear magnetic relaxation rate is studied by using the dynamical susceptibility in the nearly antiferromagnetic metals. It is obtained:

$$\frac{1}{T_{1\nu}T} = \frac{2}{N_0}\gamma_n^2 A_{hf}^2 \sum_q \frac{\mathrm{Im}\chi_\nu(Q+q,\omega_0)}{\omega_0} \tag{19}$$

Figure 1: The temperature dependence of the inverse of the reduced staggered magnetic susceptibility $y_\nu(\nu = \|, \text{ or } \perp)$ when $y_{0\|}=0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|}=y_{2\|}=3,\ y_{1\perp}=y_{2\perp}=1,$ respectively.

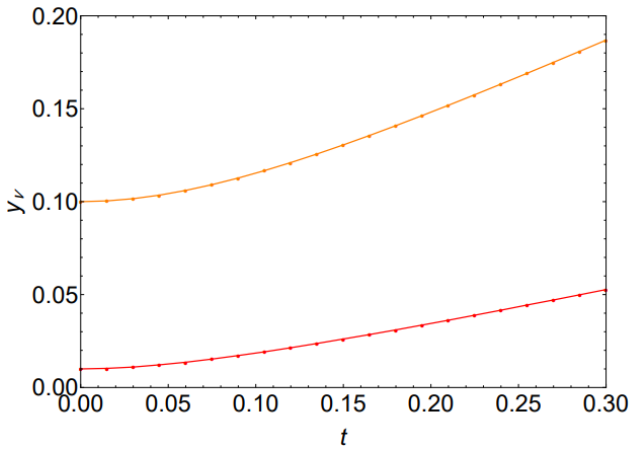


where $T_{1\nu}(\nu=\| \text{ or } \perp)$ is a nuclear magnetic relaxation time, A_{hf} is the hyperfine coupling constant. γ_n is the nuclear gyromagnetic ratio, and N_0 is the number of magnetic atoms. The nuclear magnetic relaxation rate in the nearly antiferromagnetic metal is

$$\frac{1}{T_{1\nu}t} = (g\mu_B)^2(\gamma_n A_{hf})^2 \frac{1}{2\pi\alpha_{s\nu}T_{A_\nu}}[1 - \sqrt{y}_\nu \arctan\frac{1}{\sqrt{y}_\nu}](\nu = \| \ or \perp) \tag{20}$$

where g is the g-factor of the conduction electron, and μ_B is the Bohr's magneton. Fig. 3 shows the temperature dependence of the nuclear magnetic relaxation rate with $y_{0\|}=0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|} = y_{2\|} = 3$ and $y_{1\perp} = y_{2\perp} = 1.$ Fig. 4 shows the temperature dependence of the nuclear magnetic relaxation rate with $y_{0\|}=0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|} = y_{2\|} = 1$ and $y_{1\perp} = y_{2\perp} = 3.$ .From Eq. (20), $\frac{1}{T_{1\nu}t}$ has *T*-linear dependence at low temperatures because the inverse of the staggered magnetic susceptibility has T^2-dependence at low temperatures. It has $T^{1/2}$ -linear dependence in elevated temperatures because the inverse of the staggered magnetic susceptibility has *T*-linear dependence in elevated temperatures.

Figure 2: The temperature dependence of the inverse of the reduced staggered magnetic susceptibility $y_\nu(\nu = \| \ or \ \perp)$ when $y_{0\|}$=0.01 (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|} = y_{2\|} = 1$, $y_{1\perp} = y_{2\perp} = 3.$, respectively.



The *T*-linear coefficient of the specific heat

The free energy of spin fluctuations is obtained as follows [5]:

$$F_{sf} = \int_0^{\omega_c} d\omega f(\omega) \sum_{q,\nu} \frac{1}{\pi} \frac{\Gamma_{q\nu}}{\Gamma_{q\nu}^2 + \omega^2} \tag{21}$$

with

$$f(\omega) = \frac{\omega}{2} + T\ln(1 - e^{-\omega/T}) \tag{22}$$

where $\Gamma_{q\nu}$ is the damping constant of spin fluctuations ($\nu = \|$ or $\perp$). The specific heat of spin fluctuations is

$$\frac{C_m}{T} = -\frac{\partial^2 F_{sf}}{\partial T^2}. \tag{23}$$

The *T*-linear coefficient of the specific heat γ_m is obtained

$$\gamma_m = \frac{N_0}{2T_0}(3 - \frac{1}{\sqrt{y_{0\|}}}\arctan\frac{1}{\sqrt{y_{0\|}}} - \frac{2}{\sqrt{y_{0\perp}}}\arctan\frac{1}{\sqrt{y_{0\perp}}}). \tag{24}$$

From Eq.(24), γ_m increases when $y_{0\nu}$ increases

Figure 3: The temperature dependence of $\frac{2\pi\alpha_{s\nu}T_{A_\nu}}{T_{1\nu}t(\gamma_n A_{hf})^2(g\mu_B)^2}(\nu = \| \ or \ \perp)$ when $y_{0\|}=0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|} = y_{2\|} = 3$, $y_{1\perp} = y_{2\perp} = 1.$,respectively.

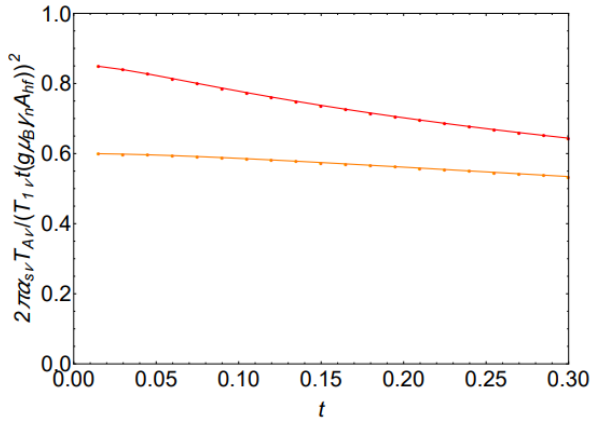
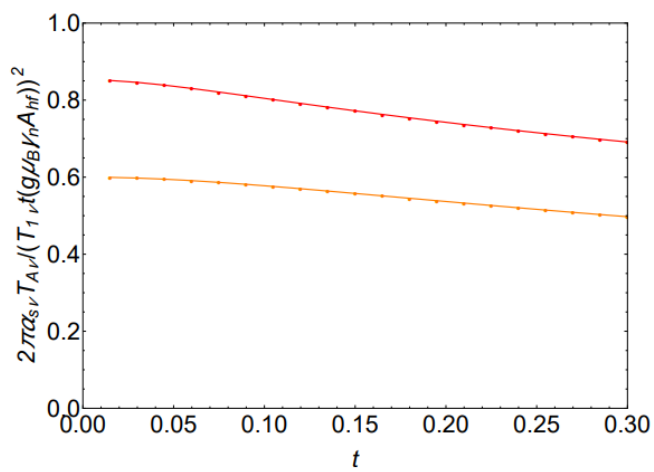


Figure 4. The temperature dependence of $\frac{2\pi\alpha_{s\nu}T_{A_\nu}}{T_{1\nu}t(\gamma_n A_{hf})^2(g\mu_B)^2}(\nu = \| \ or \ \perp)$ when $y_{0\|}=0.01$ (the red line), $y_{0\perp} = 0.1$ (the orange line), $y_{1\|} = y_{2\|} = 1$, $y_{1\perp} = y_{2\perp} = 3.$,respectively.



Conclusions

We have made the self-consistent renormalization theory of anisotropic spin fluctuations in three dimensional nearly antiferromagnetic metals beyond the random phase approximation. We have investigated the temperature dependence of the inverse of the staggered magnetic susceptibility, nuclear magnetic relaxation rate, and the *T*-linear coefficient of the specific heat in nearly antiferromagnetic metals. We have found that the temperature dependence of the inverse of the staggered magnetic susceptibility has T^2-linear behavior at low temperatures. With increasing

temperatures, it has T-linear dependence. The nuclear magnetic relaxation rate has T-linear dependence at low temperatures. With increasing temperatures, it has $T^{1/2}$-linear dependence. The anisotropy appears in the inverse of the staggered magnetic susceptibility and the nuclear magnetic relaxation rate by anisotropic spin fluctuations.

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